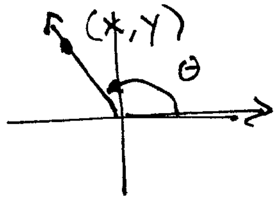
3.3 Unit circle definition of trig functions

<u>Recall</u>: (1) Ratio definition of trig functions

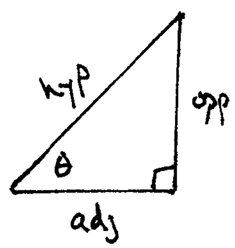


$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$ where $r = \sqrt{x^2+y^2}$

$\tan\theta = y/x$

(2) Right triangle definition

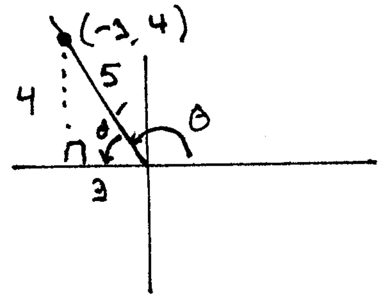


$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

$\tan\theta = \frac{opp}{adj}$

combination of (1) and (2)



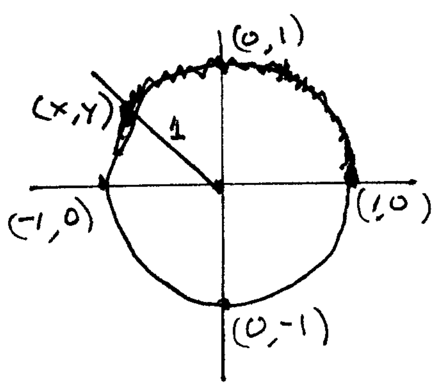
$\sin\theta = +\sin\theta' = \frac{4}{5}$

$\cos\theta = -\cos\theta' = -\frac{3}{5}$

$\tan\theta = -\tan\theta' = -\frac{4}{3}$

(3) Unit circle definition

<u>Remark</u>: we can think of this as a special case of the ratio definition where $r = \sqrt{x^2+y^2} = 1$.



Given a real number s, represent s by a directed arc around the unit circle. Let (x, y) denote the point at the end of the arc, and define

$\sin s = y$ $\csc s = \frac{1}{y}$

$\cos s = x$ $\sec s = 1/x$

$\tan s = \frac{y}{x}$ $\cot s = \frac{x}{y}$

(provided we're not dividing by zero)

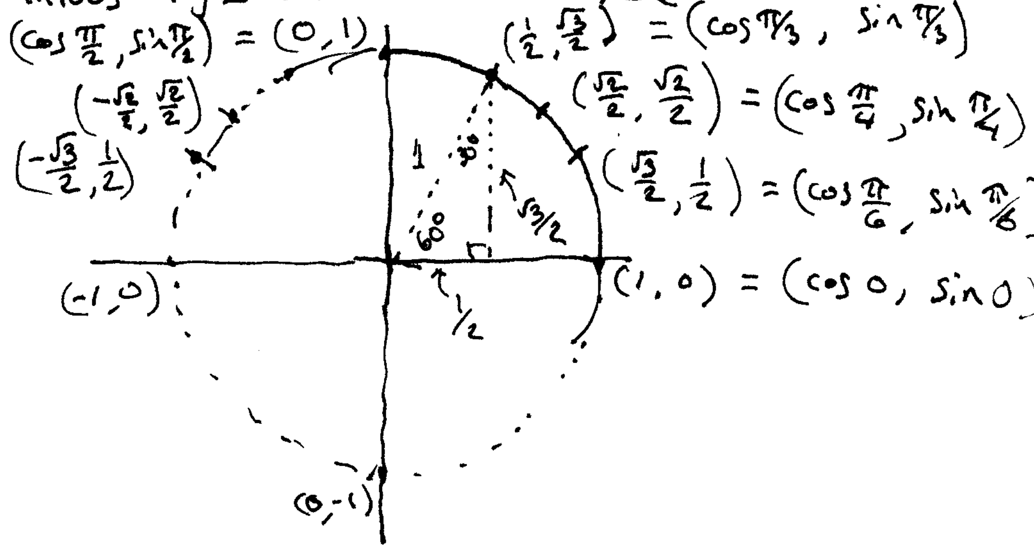
<u>Domains</u> of the "circular functions"

Sine and cosine: $(-\infty, \infty)$

tangent and secant: $\{s \mid s \neq (2n+1)\cdot\frac{\pi}{2}$, where n is any integer$\}$

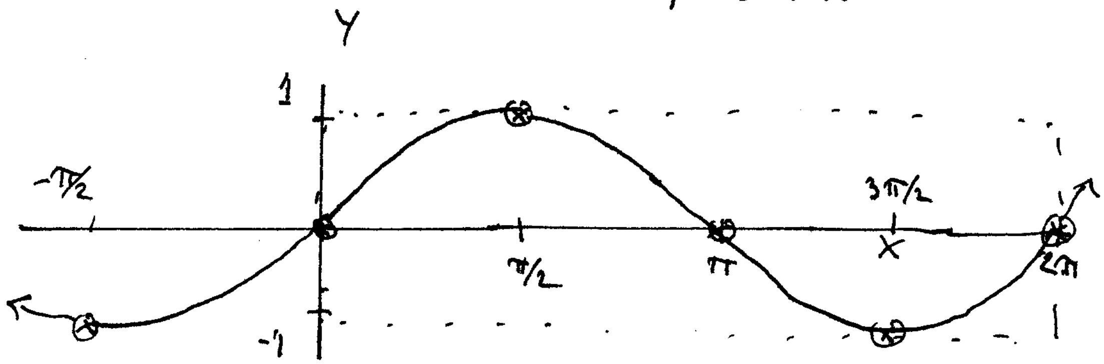
cotangent and cosecant: $\{s \mid s \neq (2n)\cdot\frac{\pi}{2}\}$
$= \{s \mid s \neq n\pi$, where is any integer$\}$

Famous angles on the unit circle



4.1 Graph of the Sine Function

$y = \sin x$



Domain $= (-\infty, \infty)$

Range $= [-1, 1]$

Sine periodic, period $= 2\pi$, that is

$$\sin(x + 2\pi) = \sin x.$$

Sine is an <u>odd</u> function, that is

$$\sin(-x) = -\sin x$$

(The graph is symmetric about the origin.)