Warm-up

ex (similar to Quiz 3 # 2)

$$y = \frac{1 - \cos 2x}{2}$$

what are i) the amplitude
ii) period
iii) phase shift
iv) vertical translation
of the graph of this equation.

First put this in the form $\boxed{y = c + a\cos b(x-d)}$ for suitable values of a, b, c and d.

$$y = \frac{1-\cos 2x}{2} = \frac{1}{2} - \frac{\cos(2x)}{2}$$

$$y = \frac{1}{2} - \frac{1}{2}\cos[2(x-0)]$$

so $a = -\frac{1}{2} \Rightarrow$ amplitude $= |a| = |-\frac{1}{2}| = \frac{1}{2}$

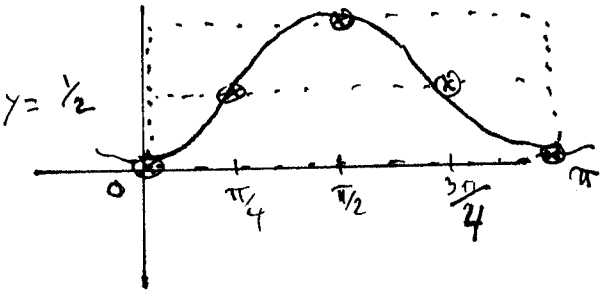
$b = 2 \Rightarrow$ period $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

$c = \frac{1}{2} \Rightarrow$ vertical translation $= +\frac{1}{2}$

$d = 0 \Rightarrow$ phase shift $= 0$

range $= [0, 1]$

min $= c - |a|$
max $= c + |a|$



Chapter 5 - Trig Identities

Remark

Equations come in three types.

(1) Conditional equations ex: $2x+3=11$

A conditional equation is true for some x but not all x.

(2) Impossible (or inconsistent) equation

ex: $x+2=x$.

An impossible equation is true for no x.

(3) Identity ex: $(x+3)^2 = x^2+6x+9$

An identity is true for all values of x.

So if $x=10$, $(10+3)^2 = 10^2+6(10)+9$

So $13^2 = 169$

5.1 Fundamental Identities

- Reciprocal identities e.g. $\sec\theta = \frac{1}{\cos\theta}$
- Quotient identities e.g. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- Pythagorean identities

$$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

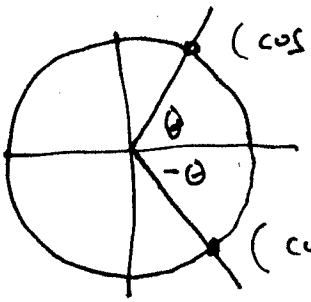
$$\tan^2\theta + 1 = \sec^2\theta$$

Likewise, $\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$$1 + \cot^2\theta = \csc^2\theta$$

Sine is an odd function:

that is $\sin(-\theta) = -\sin\theta$



← the y-coodmates are opposite, so

$\sin(-\theta) = -\sin\theta$

↖ ← the x-coodinates are equal

So

$\cos(-\theta) = \cos\theta$

→ cosine is an even function.