5.1 Fundamental identities (cont'd)

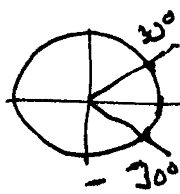
Even - Odd Identities

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

$\csc(-\theta) = -\csc\theta$ $\sec(-\theta) = \sec\theta$ $\cot(-\theta) = -\cot\theta$

[odd] ↑ [even functions] ↑ [odd functions] ↑

examples: $\sin(-30^\circ) = -\sin 30^\circ = -\frac{1}{2}$



$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

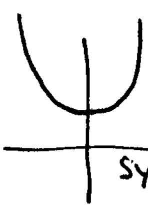
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\cos(-30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\tan(-30^\circ) = -\tan 30^\circ = -\frac{1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

Remark: Q: why the name "odd" or "even" function?

ex: $f(x) = x^2 + 1$ ← ← even exponents



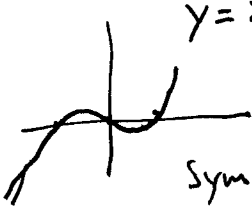
$y = x^2 + 1$

Symmetric across the y-axis

$f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x)$

$g(x) = x^3 - x$ ← odd exponents

$y = x^3 - x$



Symmetric about the origin

$g(-x) = (-x)^3 - (-x) = -x^3 + x$

$= -(x^3 - x) = -g(x)$

Write each expression in terms of sine and cosine, then simplify so that no quotients appear and all functions are of θ only.

54) $\tan\theta \cos\theta = \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{1}$ Quotient identity

$= \frac{\sin\theta}{1} = \sin\theta$

60) $\cot^2\theta \, (1 + \tan^2\theta)$

$= \frac{\cos^2\theta}{\sin^2\theta}\left(1 + \frac{\sin^2\theta}{\cos^2\theta}\right)$ Quotient identities

$= \frac{\cos^2\theta}{\sin^2\theta} + \frac{\cancel{\cos^2\theta}}{\cancel{\sin^2\theta}} \, \frac{\cancel{\sin^2\theta}}{\cancel{\cos^2\theta}}$

$= \frac{\cos^2\theta}{\sin^2\theta} + 1 = \frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta}$

$= \frac{1}{\sin^2\theta}$ Pythagorean ID

$= \csc^2\theta$ Reciprocal identity

5.2 Verifying Trig Identities

Remark: Not only can we use all the Fundamental Identities, but also equations which are equivalent to the fundamental identities:

ex: Not only $\sin^2 x + \cos^2 x = 1$

but also $\cos^2 x = 1 - \sin^2 x$

or $\sin^2 x = 1 - \cos^2 x$

Likewise $\tan^2 x + 1 = \sec^2 x$

but also $\tan^2 x = \sec^2 x - 1$

50) verify the identity.

$\sin^2\beta\,(1 + \cot^2\beta) = 1$

$$
\begin{aligned}
\sin^2\beta\,(1+\cot^2\beta) &= \sin^2\beta + \sin^2\beta\cot^2\beta && \text{Distributed} \\
&= \sin^2\beta + \sin^2\beta \cdot \frac{\cos^2\beta}{\sin^2\beta} && \text{Quotient ID} \\
&= \sin^2\beta + \cos^2\beta && \text{Canceled the } \sin^2\beta \\
&= 1 && \text{pythagorean identity}
\end{aligned}
$$

Alternatively:

$$
\begin{aligned}
\sin^2\beta\,(1+\cot^2\beta) &= \sin^2\beta\csc^2\beta && \text{pythagorean ID} \\
&= \sin^2\beta \cdot \frac{1}{\sin^2\beta} && \text{reciprocal ID} \\
&= 1 && \text{cancel}
\end{aligned}
$$