5.2 Verifying identities (cont'd)

62) verify:

$$\frac{1}{\sec\alpha - \tan\alpha} = \sec\alpha + \tan\alpha$$

$$\frac{1}{\sec\alpha - \tan\alpha} = \frac{1}{\frac{1}{\cos\alpha} - \frac{\sin\alpha}{\cos\alpha}}$$ Reciprocal, and quotient identities.

$$= \frac{1}{\frac{1-\sin\alpha}{\cos\alpha}}$$ Subtract fractions.

$$= \frac{\cos\alpha}{1-\sin\alpha}$$ Take reciprocal

$$= \frac{\cos\alpha}{1-\sin\alpha} \cdot \frac{1+\sin\alpha}{1+\sin\alpha}$$ ← TRICK: Equivalent fraction.

$$= \frac{\cos\alpha(1+\sin\alpha)}{1-\sin^2\alpha}$$ Difference of squares multiplication

$$= \frac{\cos\alpha(1+\sin\alpha)}{\cos^2\alpha}$$ Pythagorean identity.

$$= \frac{1+\sin\alpha}{\cos\alpha}$$ Cancel one $\cos\alpha$.

$$= \frac{1}{\cos\alpha} + \frac{\sin\alpha}{\cos\alpha}$$ Separate fractions.

$$= \sec\alpha + \tan\alpha$$ Reciprocal and quotient identities

5.3 Sum and Difference Identities for Cosine

$$\boxed{\begin{aligned} \cos(A+B) &= \cos A \cos B - \sin A \sin B \\ \cos(A-B) &= \cos A \cos B + \sin A \sin B \end{aligned}}$$

sum formula...
difference formula for cosine

<u>examples</u> Find the <u>exact</u> value of $\cos 75°$.

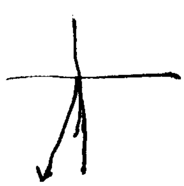
<u>Hint:</u> $75° = 45° + 30°$

$$\begin{aligned} \cos 75° &= \cos(45° + 30°) \\ &= \cos 45° \cos 30° - \sin 45° \sin 30° \\ &= \left(\frac{\sqrt{2}}{2}\right)\cdot\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4} \end{aligned}$$

<u>ex</u>: $\cos \frac{17\pi}{12} = ?$

<u>Hint</u>: $\frac{8\pi}{12} + \frac{9\pi}{12} = \frac{17\pi}{12}$

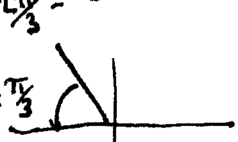
$= \frac{2\pi}{3} + \frac{3\pi}{4}$



$$\begin{aligned} \cos \frac{17\pi}{12} &= \cos \frac{2\pi}{3} \cos \frac{3\pi}{4} - \sin \frac{2\pi}{3} \sin \frac{3\pi}{4} \\ &= \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = -\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right) \end{aligned}$$

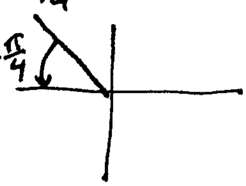
$\frac{2\pi}{3} = 120°$

$60° = \frac{\pi}{3}$



$3\pi/4 = 135°$

$45° = \frac{\pi}{4}$



ex: Find the exact value of

$$\cos 173^\circ \cos 83^\circ + \sin 173^\circ \sin 83^\circ$$
$$= \cos(173^\circ - 83^\circ) = \cos 90^\circ = 0$$

Cofunction identities

$$\cos(90^\circ - \theta) = \sin\theta \qquad \cot(90^\circ - \theta) = \tan\theta$$
$$\sin(90^\circ - \theta) = \cos\theta \qquad \sec(90^\circ - \theta) = \csc\theta$$
$$\tan(90^\circ - \theta) = \cot\theta \qquad \csc(90^\circ - \theta) = \sec\theta$$

Added after class:

Remark ① We get the first of these identities by taking $A = 90^\circ$ and $B = \theta$ in the difference identity for cosine:

$$\cos(90^\circ - \theta) = \cos 90^\circ \cos\theta + \sin 90^\circ \sin\theta$$
$$= (0)(\cos\theta) + (1)(\sin\theta)$$
$$= \sin\theta$$

② Leave a gap where θ is in the above identity:

$$\cos[90^\circ - (\quad)] = \sin(\quad)$$

Then, in the gap, write $90^\circ - \theta$:

$$\cos[90^\circ - (90^\circ - \theta)] = \sin(90^\circ - \theta)$$

This simplifies to:

$$\cos(90^\circ - 90^\circ + \theta) = \sin(90^\circ - \theta)$$

or:

$$\cos\theta = \sin(90^\circ - \theta)$$

That is, we've derived the second cofunction identity.

$$\sin(90^\circ - \theta) = \cos\theta.$$