5.4 Sum and Difference Identities for Sine and Tangent

$$\boxed{\begin{aligned} \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ \sin(A-B) &= \sin A \cos B - \cos A \sin B \end{aligned}}$$

sum formula
difference formula for sine

ex: Find the exact value of $\sin 75^\circ$.

Note: $75^\circ = 45^\circ + 30^\circ$

$$\begin{aligned} \sin 75^\circ &= \sin(45^\circ + 30^\circ) \\ &= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4} \end{aligned}$$

ex: 26) Find the exact value of

$$\begin{aligned} &\sin 40^\circ \cos 50^\circ + \cos 40^\circ \sin 50^\circ \\ &= \sin(40^\circ + 50^\circ) = \sin 90^\circ = 1 \end{aligned}$$

38) Rewrite $\sin(45^\circ + \theta)$ as an expression involving functions of θ alone.

$$\begin{aligned} \sin(45^\circ + \theta) &= \sin 45^\circ \cos\theta + \cos 45^\circ \sin\theta \\ &= \frac{\sqrt{2}}{2}\cos\theta + \frac{\sqrt{2}}{2}\sin\theta \end{aligned}$$

M 170 17.04.13

$$\boxed{\begin{aligned} \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \end{aligned}}$$

Sum and difference formulas for tangent

ex: Find the exact value of $\tan \frac{13\pi}{12}$.

Note: $\frac{13\pi}{12} = \frac{9\pi}{12} + \frac{4\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{3}$

$\frac{3\pi}{4}$: $135°$ so ref. angle $= 45°$

$\frac{\pi}{3}$: $60°$

$$\tan \frac{13\pi}{12} = \tan\left(\frac{3\pi}{4} + \frac{\pi}{3}\right)$$

$$= \frac{\tan \frac{3\pi}{4} + \tan \frac{\pi}{3}}{1 - \tan \frac{3\pi}{4} \tan \frac{\pi}{3}}$$

$$= \frac{(-1) + (\sqrt{3})}{1 - (-1)\cdot(\sqrt{3})} = \frac{-1 + \sqrt{3}}{1 + \sqrt{3}}$$

Note: we could have also used

$$= \frac{13\pi}{12} = 150° + 45° = \frac{5\pi}{6} + \frac{\pi}{4}$$

Note: We also could have used

$$\tan \frac{13\pi}{12} = \frac{\sin \frac{13\pi}{12}}{\cos \frac{13\pi}{12}}$$

52) a) Given: $\sin s = \frac{3}{5}$, s is in Q I

$\sin t = -\frac{12}{13}$, t is in Q III

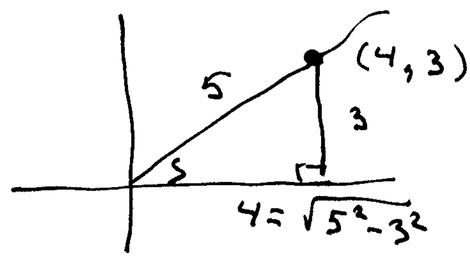
Problem: Find $\sin(s+t)$.

Use: $\sin(s+t) = \sin s \cos t + \cos s \sin t$

know is $3/5$ know is $\frac{-12}{13}$

What is $\cos s$?

Know $\sin s = \frac{\text{opp}}{\text{hyp}} = \frac{3}{5}$



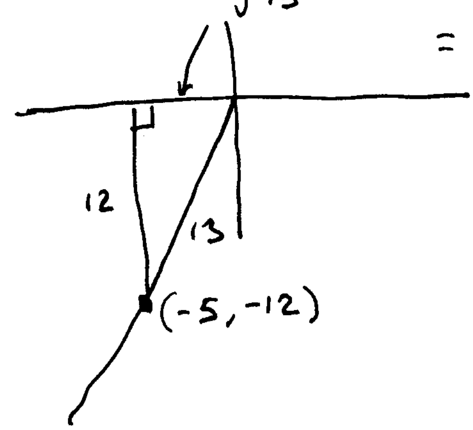
So $\cos s = \frac{\text{adj}}{\text{hyp}} = \frac{4}{5}$

$\sqrt{13^2 - 12^2} = \sqrt{169 - 144} = \sqrt{25} = 5$

What is $\cos t$?

Know $\sin t = \frac{\text{opp}}{\text{hyp}} = \frac{-12}{13}$

$\cos t = -\frac{5}{13}$



Answer: $\sin(s+t) = \sin s \cos t + \cos s \sin t$

$= \left(\frac{3}{5}\right)\left(\frac{-5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{-12}{13}\right) = \frac{-15}{65} + \frac{-48}{65}$

$= \boxed{-\frac{63}{65}}$