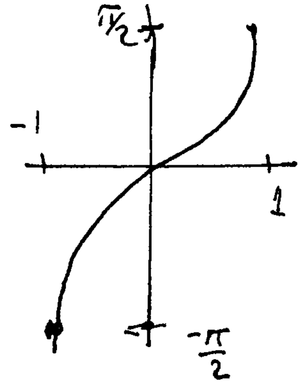
Math 170 27 APR 2017

6.1 Inverse trig functions (cont'd)



$y = \sin^{-1} x$

<u>Defn</u>: When we say $y = \sin^{-1} x$

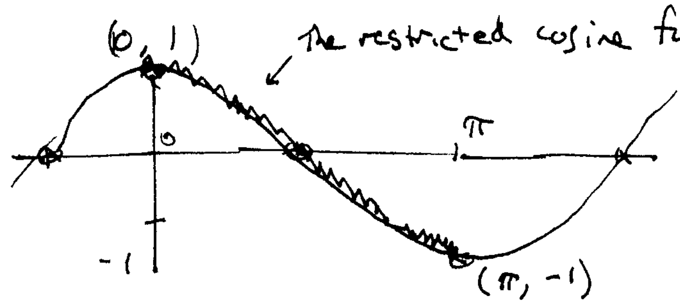
we mean ① $\sin y = x$

and ② $-\pi/2 \le y \le \pi/2$ ← memorize this:

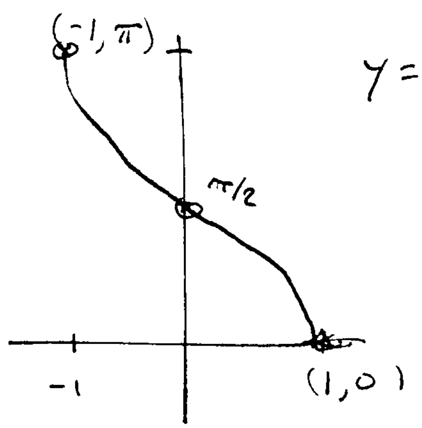
The range of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

<u>Remark</u>: $y = \cos x$ is not one-to-one

The restricted cosine function <u>is</u> one-to-one



Restrict to $[0, \pi]$



$y = \cos^{-1} x$

Domain $= [-1, 1]$
Range $= [0, \pi]$

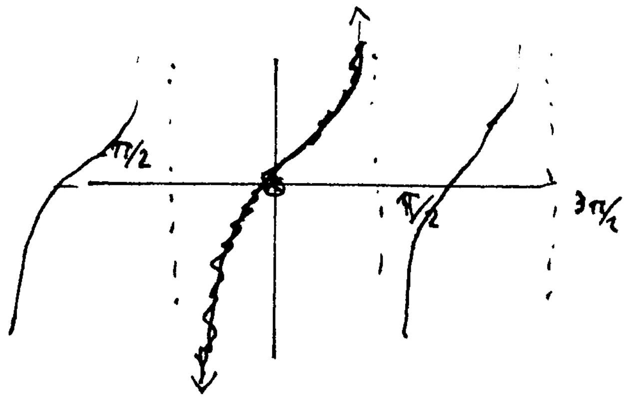
<u>Defn</u>: $y = \cos^{-1} x$ means

① $\cos y = x$ and

② $0 \le y \le \pi$

<u>Informally</u> $\cos^{-1} x$ is the angle whose cosine is x, but choose the angle in Q I or Q II.

Remark: $y = \tan x$ is not one-to-one

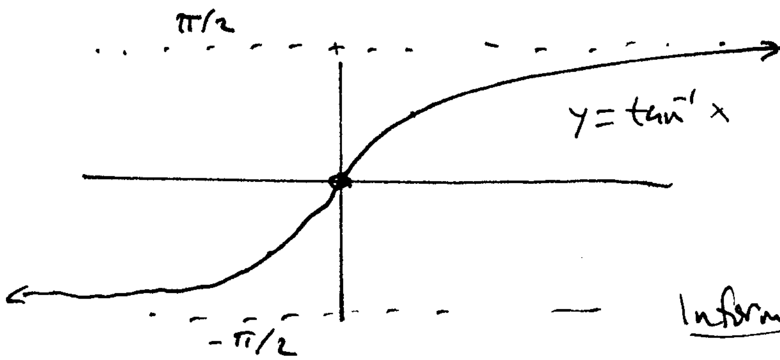


Restrict the domain to
$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$
to make the restricted function one-to-one.

Defn: $y = \tan^{-1} x$ [or $y = \arctan x$]

means ① $\tan y = x$

and ② $-\frac{\pi}{2} < y < \frac{\pi}{2}$ ← Memorize



Domain $= (-\infty, \infty)$

Range $= \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

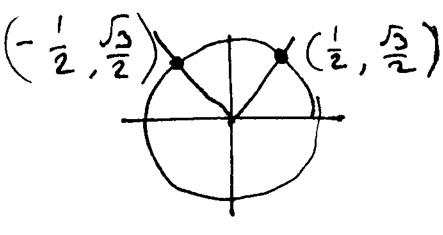
Informally $y = \tan^{-1} x$ is the angle in QI or QIV whose tangent is x.

ex: Without a calculator

a) Evaluate $\arcsin \frac{\sqrt{3}}{2}$.

$y = \arcsin \frac{\sqrt{3}}{2}$ means ① $\sin y = \frac{\sqrt{3}}{2}$

and ② $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



There are infinitely many solutions to ①.
But only one satisfies ②

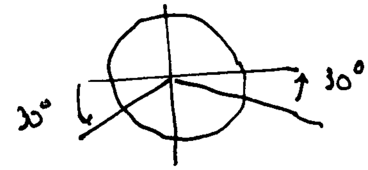
so $y = 60° = \boxed{\frac{\pi}{3}}$

ex b) Find $y = \sin^{-1}\left(-\frac{1}{2}\right)$. This says:

① $\sin y = -\frac{1}{2}$ and

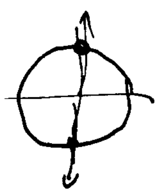
② $-\frac{\pi}{2} \le y \le \pi/2$

↑ want QI or QIV



Answer: $y = -\frac{\pi}{6}$

ex a) Find $y = \arccos 0$. This says

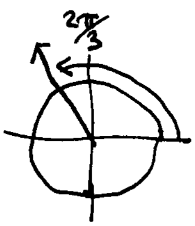


① $\cos y = 0$ and

② $0 \le y \le \pi$

Answer. $y = \frac{\pi}{2}$.

b) $y = \arccos\left(-1/2\right)$ so

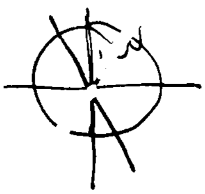


① $\cos y = -\frac{1}{2}$ and ② $0 \le y \le \pi$

so $y = \frac{2\pi}{3}$ That is $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$.

ex: Find $y = \tan^{-1}(-\sqrt{3})$ without a calculator.

so ① $\tan y = -\sqrt{3}$ and ② $-\pi/2 < y < \pi/2$

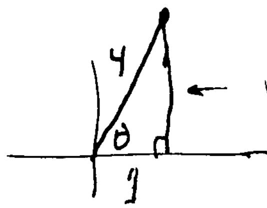


so $y = -\frac{\pi}{3}$

76) Find $\sin\left(\arccos \frac{1}{4}\right)$ without a calculator.

Let $\theta = \arccos \frac{1}{4}$. Then

$\cos\theta = \frac{1}{4}$ and θ is QI or ~~QII~~



$\leftarrow \sqrt{4^2 - 1^2} = \sqrt{15}$

$$\sin\theta = \frac{opp}{hyp} = \boxed{\frac{\sqrt{15}}{4}}$$

80) Find $\cos\left(2 \sin^{-1} \frac{1}{4}\right)$.

Let $\sin^{-1}\frac{1}{4} = \theta$ so $\sin\theta = \frac{1}{4}$ and θ is in QI.

$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$= 1 - 2(\sin\theta)^2$$
$$= 1 - 2\left(\frac{1}{4}\right)^2 = 1 - 2\cdot\frac{1}{16} = 1 - \frac{1}{8}$$
$$= \boxed{\frac{7}{8}}$$