Math 170 Tue 2 May 2017

6.2 Trig Equations I

ex: a) Solve $\sqrt{2}\cos\theta - 1 = 0$ over the interval $[0°, 360°)$.

b) ... for all solutions

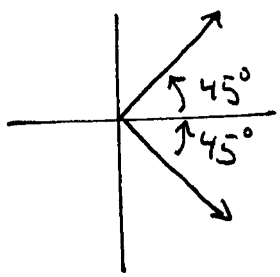
$$\sqrt{2}\cos\theta - 1 = 0$$
$$\sqrt{2}\cos\theta = 1$$
$$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

what is the reference angle?

$\theta' = 45°$

where is $\cos\theta$ positive?

QI and QIV



Answer to a) $\theta = \boxed{45°}$ or $\theta = 360° - 45° = \boxed{315°}$.

b) All solutions: $\boxed{\begin{array}{l} \theta = 45° + 360° n \\ \theta = 315° + 360° n \end{array}}$ or where n is any integer.

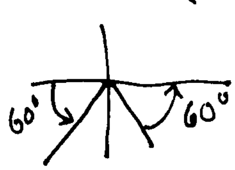
(For example, if $n = -1$, $\theta = 315° + 360°(-1) = -45°$)

ex: $\sin\theta = -\frac{\sqrt{3}}{2}$

a) Solve for $[0°, 360°)$

{Reference angle? For what θ' is $\sin\theta' = \frac{\sqrt{3}}{2}$? $\theta' = 60°$
{In which quadrants is $\sin\theta$ negative? QIII, QIV.

$$\theta = 180° + 60° = \boxed{240°}$$ or
$$\theta = 360° - 60° = \boxed{300°}$$



b) All solutions

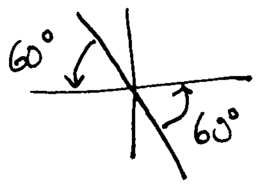
$\boxed{\begin{array}{l} \theta = 240° + 360° n \text{ or} \\ \theta = 300° + 360° n \end{array}}$

ex: Solve $\tan\theta = -\sqrt{3}$

a) .. over $[0°, 360°)$

Reference angle? $\tan\theta' = \sqrt{3}$ if $\theta' = 60°$

Quadrants where tan is negative? Q II, Q IV.



$\theta = 180° - 60° = 120°$ or

$\theta = 360° - 60° = 300°$

b) ... for all solutions $\theta = 120° + 360° n$ or

$\theta = 300° + 360° n$

Note: combined, this is $\theta = 120° + 180° n$.
[optional]

ex [Solving by factoring] Solve over $[0°, 360°)$.

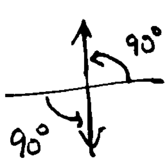
$\cos\theta \cot\theta = -\cos\theta$ Add $\cos\theta$ to both sides

$+\cos\theta \qquad +\cos\theta$

$\cos\theta\cot\theta + \cos\theta = 0$ Factor out $\cos\theta$:

$\cos\theta(\cot\theta + 1) = 0$

$\cos\theta = 0$ OR $\cot\theta + 1 = 0$



$\theta = \boxed{90°}$

or $\theta = \boxed{270°}$

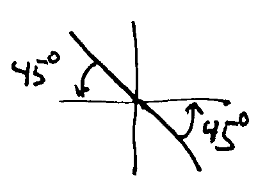
$\cot\theta = -1$ Take reciprocal of both sides:

$\tan\theta = \frac{1}{-1}$

$\tan\theta = -1$

$\tan\theta' = 1$ so $\theta' = 45°$

tan is negative in Q II or IV



$\theta = \boxed{135°}$ or $\theta = \boxed{315°}$

ex: $3\sin^2 x - \sin x - 2 = 0$ over $[0, 2\pi)$

Can we factor this? It's hard to see,

So let $u = \sin x$ (temporarily):

$$3u^2 - u - 2 = 0$$ Yes, we can factor this

Scratch work:

(1) $ac = 3(-2)$
$= -6$

(2)

p, q	$p+q$
$-6, 1$	-5
$-3, 2$	-1

(3) Split the middle term:

$$3u^2 - 3u + 2u - 2 = 0$$

(4) Factor by grouping:

$$(3u^2 - 3u) + (2u - 2) = 0$$

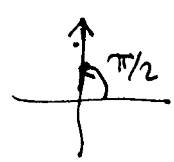
$$3u(u-1) + 2(u-1) = 0$$

$$(3u+2)(u-1) = 0$$

Replace u with sin x: $(3\sin x + 2)(\sin x - 1) = 0$

$3\sin x + 2 = 0$ OR $\sin x - 1 = 0$

$\sin x = 1$



$x = \boxed{\frac{\pi}{2}}$ $[90°]$

≈ 1.57

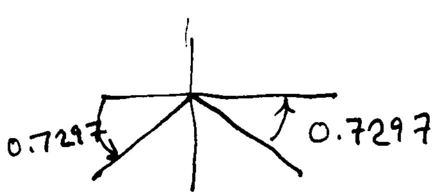
$3\sin x = -2$

$\sin x = -\frac{2}{3}$

Reference angle? $\sin\theta' = \frac{2}{3}$

So $\theta' = \sin^{-1}\frac{2}{3} = 0.7297$

Quadrants? QIII or QIV



$x = \pi + 0.7297 = \boxed{3.8713}$

OR $x = 2\pi - 0.7297 = \boxed{5.5535}$