6.3 Trig equations II

ex: Solve $2\cos\frac{x}{2} - \sqrt{2} = 0$

a) over $[0, 2\pi)$ b) all solutions

outline
- Step 1: Solve for cosine.
- Step 2: Solve for the angle.
- Step 3: Solve for x.

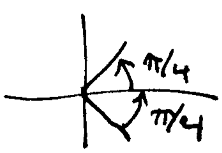
Step 1: $2\cos\frac{x}{2} = \sqrt{2}$

$$\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$$

Step 2: Let $u = \frac{x}{2}$ (temporarily), so u = angle.

$$\cos u = \frac{\sqrt{2}}{2}$$

Reference angle $= 45^\circ = \frac{\pi}{4}$ because $\cos 45^\circ = \frac{\sqrt{2}}{2}$.
Also, cosine is positive in QI and QIV.



So $u = \pi/4$ or $u = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

All solutions (angles): $u = \frac{\pi}{4} + 2\pi n$ or $u = \frac{7\pi}{4} + 2\pi n$.

Step 3: $\frac{x}{2} = \frac{\pi}{4}$ or $\frac{x}{2} = \frac{7\pi}{4}$

More generally: $\frac{x}{2} = \frac{\pi}{4} + 2\pi n$ or $\frac{x}{2} = \frac{7\pi}{4} + 2\pi n$

Solve for x: $2\left(\frac{x}{2}\right) = 2\left(\frac{\pi}{4} + 2\pi n\right)$ OR $2\left(\frac{x}{2}\right) = 2\left(\frac{7\pi}{4} + 2\pi n\right)$

Answer b) All solutions: $\boxed{x = \frac{\pi}{2} + 4\pi n \text{ OR } x = \frac{7\pi}{2} + 4\pi n}$

Answer to a) $\boxed{x = \frac{\pi}{2}}$ $\cancel{x = \frac{7\pi}{2}}$ $x = \frac{\pi}{2} \cancel{+ 4\pi = \frac{9\pi}{2}}$

$x = \cancel{\frac{7\pi}{2} + \frac{8\pi}{2} = \frac{15\pi}{2}}$

ex Solve $\cos 2x = \sin x$ over $[0, 2\pi)$

Use: $\cos 2x = 1 - 2\sin^2 x$ and substitute.

$$1 - 2\sin^2 x = \sin x$$

$$0 = 2\sin^2 x + \sin x - 1$$

Let $u = \sin x$ temporarily.

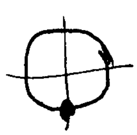
$$2u^2 + u - 1 = 0$$

$$(u + 1)(2u - 1) = 0$$

$$(\sin x + 1)(2\sin x - 1) = 0$$

Use the zero-factor property

$\sin x + 1 = 0$ or $2\sin x - 1 = 0$

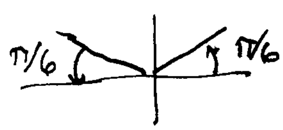


$\sin x = -1$

$\boxed{x = \frac{3\pi}{2}}$

$\sin x = \frac{1}{2}$

reference angle $= \frac{\pi}{6}$
$\sin x$ is positive in QI and QII



$\boxed{\begin{aligned} x &= \pi/6 \\ x &= 5\pi/6 \end{aligned}}$ or $(= \pi - \pi/6)$

ex: Solve $2\cos^2\theta - 2\sin^2\theta + 1 = 0$

a) over $[0°, 360°)$ b) all solutions

Use identities: Namely, $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$2(\cos^2\theta - \sin^2\theta) + 1 = 0$$

$$2\cos 2\theta + 1 = 0$$

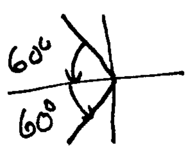
Step 1: $2\cos 2\theta = -1$

$$\cos 2\theta = -1/2$$

Step 2: Let $u = 2\theta$

$$\cos u = -\frac{1}{2}$$

$\left\{ \begin{array}{l} \text{Reference angle? } \cos u' = \frac{1}{2} \text{ if } u' = 60° \\ \cos u \text{ is negative in QII and QIII.} \end{array} \right.$



$u = 180° - 60° = 120°$

or $u = 180° + 60° = 240°$

General answer: $u = 120° + 360° n$

or $u = 240° + 360° n$

Step 3: $2\theta = 120° + 360° n$

or $2\theta = 240° + 360° n$

Multiply both sides by $\frac{1}{2}$:

$\theta = \frac{1}{2}(120° + 360° n) = 60° + 180° n$

or $\theta = \frac{1}{2}(240° + 360° n) = 120° + 180° n$

Which of these are in $[0°, 360°)$?

$$\{60°, 120°, 240°, 300°\}$$

52) $\sin\frac{x}{2} + \cos\frac{x}{2} = 1$ Solve over $[0, 2\pi)$

Replace $\cos\frac{x}{2}$ with $\pm\sqrt{1-\sin^2\frac{x}{2}}$

$$\sin\frac{x}{2} \pm \sqrt{1-\sin^2 x/2} = 1$$

$$\pm\sqrt{1-\sin^2\frac{x}{2}} = 1 - \sin\frac{x}{2}$$

Square both sides <u>BUT</u> remember this <u>may</u> introduce extraneous solutions.

$$\left(\sqrt{1-\sin^2\frac{x}{2}}\right)^2 = \left(1-\sin\frac{x}{2}\right)^2$$

$$\cancel{1} - \sin^2\frac{x}{2} = \cancel{1} - 2\sin\frac{x}{2} + \sin^2\frac{x}{2}$$

$$0 = 2\sin^2\frac{x}{2} - 2\sin\frac{x}{2}$$

$$0 = 2\sin\frac{x}{2}\left(\sin\frac{x}{2} - 1\right)$$

$\sin\frac{x}{2} = 0$ or $\sin\frac{x}{2} - 1 = 0$

$\sin u = 0$

$\frac{x}{2} = u = 0, \pi, 2\pi, 3\pi \ldots$

$\boxed{x = 0}, \cancel{2\pi}, \cancel{4\pi}, \cancel{6\pi}$

$\sin\frac{x}{2} = 1$

$\sin u = 1$

$\frac{x}{2} = u = \frac{\pi}{2}, \ldots$

$\boxed{x = \pi}$

Extraneous?

<u>Check</u>: $\sin\frac{0}{2} + \cos\frac{0}{2} \overset{?}{=} 1$

$0 + 1 \overset{\checkmark}{=} 1$

<u>Check</u>: $\sin\frac{\pi}{2} + \cos\frac{\pi}{2} \overset{?}{=} 1$

$1 + 0 \overset{\checkmark}{=} 1$