Math 170   May 9, 2017

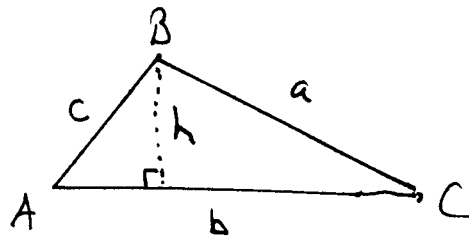# 7.1 Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ OR $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Why? Use $\frac{opp}{hyp} : \frac{h}{c} = \sin A$. Also $\frac{h}{a} = \sin C$

So $h = c \sin A = a \sin C$

So $\dfrac{\cancel{a} \sin A}{a \cancel{b}} = \dfrac{\cancel{a} \sin C}{\cancel{a} c}$ ⇒ $\dfrac{\sin A}{a} = \dfrac{\sin C}{c}$

ex [SAA] Solve triangle ABC if $A = 28.8°$, $C = 102.6°$ and $c = 25.3$ in.

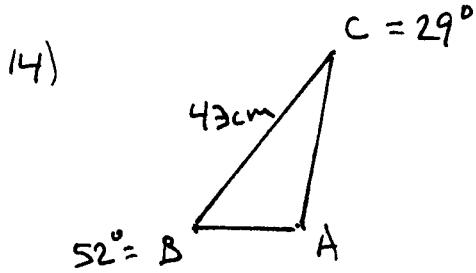Use $\dfrac{a}{\sin A} = \dfrac{c}{\sin C}$: $\dfrac{a}{\sin 28.8°} = \dfrac{25.3 \text{ in}}{\sin 102.6°}$

$a = \dfrac{25.3 \sin 28.8°}{\sin 102.6°} = \dfrac{25.3 (0.48175)}{0.97592} = 12.4891 \approx \boxed{12.5 \text{ in}}$

$B = 180° - A - C = 180° - 28.8° - 102.6° = \boxed{48.6°}$

Use $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ ⇒ $b = \dfrac{c \sin B}{\sin C} = \dfrac{25.3 \sin 48.6°}{\sin 102.6°} = \dfrac{25.3(.7501)}{0.97592}$

$= 19.4461 \approx \boxed{19.4 \text{ in}}$

14)

C = 29°
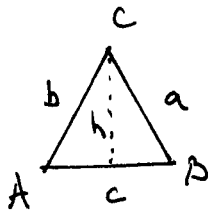
43cm

52° = B   A

$A = 180° - 52° - 29° = \boxed{99°}$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Solve for b

$$\Rightarrow \quad b = \frac{a \sin B}{\sin A} = \frac{43 \sin 52°}{\sin 99°} = \boxed{34 \text{ cm}}$$

$$\frac{c}{\sin C} = \frac{a}{\sin A} \quad \Rightarrow \quad C = \frac{a \sin C}{\sin A} = \frac{43 \sin 29°}{\sin 99°} = \boxed{21 \text{ cm}}$$

Area of a triangle

C

b   h   a

A   c   B

$$\text{Area} = \frac{1}{2}(\text{base})(\text{ht}) = \frac{1}{2} \cdot c \cdot h$$

$$= \frac{1}{2} \cdot c \cdot b \sin A$$

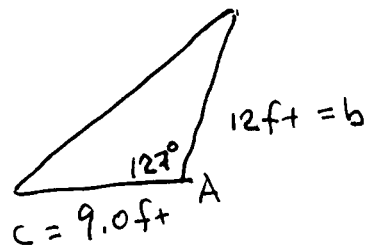$$\boxed{\text{Area} = \frac{1}{2} bc \sin A}$$

$$= \frac{1}{2} ac \sin B = \frac{1}{2} ab \sin C$$

because

$$\frac{h}{b} = \frac{opp}{hyp} = \sin A$$

ex:

127°

12ft = b

c = 9.0ft   A

Find the area

$$\text{Area} = \frac{1}{2} bc \sin A$$

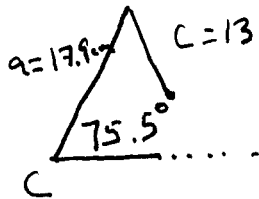$$= \frac{1}{2}(12)(9.0)\sin 127°$$

$$= \boxed{43 \text{ ft}^2}$$

7.2   SSA case (ambiguous)

ex: (Try to) solve triangle ABC if
$$a = 17.9 \text{ cm}, \quad c = 13.2 \text{ cm} \quad \text{and} \quad C = 75.5°$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow \sin A = \frac{a \sin C}{c}$$

$$= \frac{(17.9 \text{ cm}) \sin 75.5°}{13.2 \text{ cm}}$$

C = 13.2 cm is too short to reach the base.

$$\sin A = 1.3129 \longleftarrow \text{Hey!}$$

This is bigger than 1. It cannot be the sine of ANY angle.

No Solution