Math 252 1 May 2018

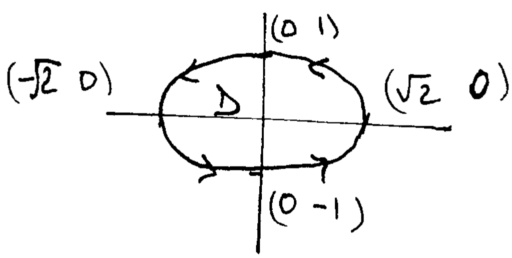
16.4 Green's Thm [additional notes]

8) Use Green's Theorem

$$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$$

to evaluate $\displaystyle\int_C y^4\,dx + 2xy^3\,dy = (*)$

where C is the ellipse $x^2+2y^2=2$, positively oriented.



$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{\partial}{\partial x}\left[2xy^3\right] - \frac{\partial}{\partial y}\left[y^4\right]$$

$$= 2y^3 - 4y^3 = -2y^3$$

By Green's Theorem,

$$(*) = \iint_D -2y^3\,dy\,dx$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{1-\frac{1}{2}x^2}}^{\sqrt{1-\frac{1}{2}x^2}} -2y^3\,dy\,dx$$

Boundaries of D:

$x^2+2y^2=2$

$\Rightarrow \quad 2y^2 = 2 - x^2$

$y^2 = 1 - \frac{1}{2}x^2$

$y = \pm\sqrt{1-\frac{1}{2}x^2}$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \left[-\frac{1}{2}y^4\right]_{-\sqrt{1-\frac{1}{2}x^2}}^{\sqrt{1-\frac{1}{2}x^2}} dx$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \left[-\frac{1}{2}\left(1-\frac{1}{2}x^2\right)^2 + \left(1-\frac{1}{2}x^2\right)^2\right] dx$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} 0\,dx = \boxed{0}$$

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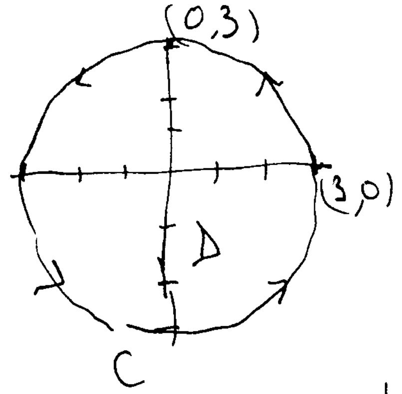
ex:



a) Calculate $\int_C x\,dy$

where C has parametorization: $x = 3\cos t$, $y = 3\sin t$, $0 \le t \le 2\pi$

so $dx = -3\sin t\,dt$, $dy = 3\cos t\,dt$

b) Explain why this calculates the area inside the circle.

a) $$\int_C x\,dy = \int_0^{2\pi} (3\cos t)(3\cos t)\,dt$$

$$= 9\int_0^{2\pi} \cos^2 t\,dt = 9\int_0^{2\pi} \left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2t\right) dt$$

$$= 9\left[\tfrac{1}{2}t + \tfrac{1}{4}\sin 2t\right]_0^{2\pi} = 9\left[\tfrac{1}{2}(2\pi) - \tfrac{1}{2}(0)\right]$$

$$= \boxed{9\pi} = (\text{area of inside a circle of radius } 3) = \pi \cdot 3^2$$

b) why is this calculating area?

$$\int_C 0\,dx + x\,dy = \iint_D \left[\frac{\partial(x)}{\partial x} - \frac{\partial(0)}{\partial y}\right] dA$$

$$= \iint_D 1\,dA = \iint_D dA = \text{area of } D.$$

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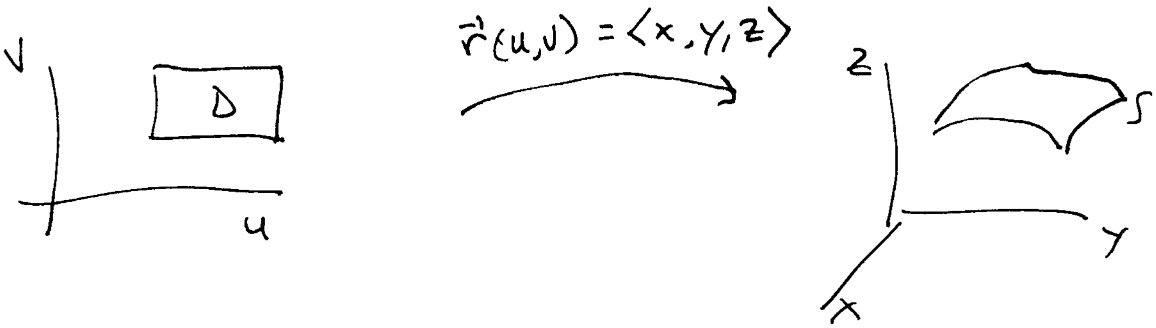
16.7 Surface Integrals [follows notes on 16.6 Surface Area of parametric Surfaces]

Idea of a surface integral of $f(x,y,z)$

$$\iint_S f(x,y,z)\,dS \approx \sum_i \sum_j f(x_{ij}, y_{ij}, z_{ij})\,\Delta S$$

g/cm^2 · cm^2



$$\iint_S f(x,y,z)\,dS = \iint_D f(x(u,v), y(u,v), z(u,v))\,|r_u \times r_v|\,du\,dv$$