Wed 27 JAN 2016

Math 281

12.1 Distance formula in space

Distance between (x_1, y_1, z_1) and (x_2, y_2, z_2) is

$$\text{Distance} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

ex Used "backwards" to say what we can about points (x, y, z) which lie 7 units from the point $(4, -8, 1)$.

$$7 = \sqrt{(x-4)^2 + (y+8)^2 + (z-1)^2}$$

Equivalent: $(x-4)^2 + (y+8)^2 + (z-1)^2 = 49$

the equation of a sphere, radius 7, center $(4, -8, 1)$.

ex: $(x-3)^2 + (y+5)^2 + z^2 = 9$ is the equation of a sphere, center $= (3, -5, 0)$ and radius 3.

ex 16) $x^2 + y^2 + z^2 + 8x - 6y + 2z + 17 = 0$

$x^2 + 8x + 16 + y^2 - 6y + 9 + z^2 + 2z + 1 = -17 + 16 + 9 + 1$

$(x+4)^2 + (y-3)^2 + (z+1)^2 = 9$

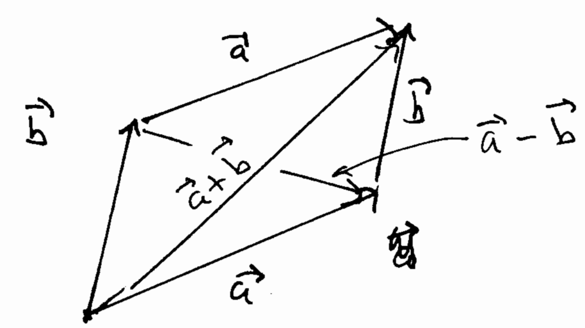
center $= (-4, 3, -1)$ radius $= 3$

29) Describe in words the region in $\mathbb{R}^3$ represented by $\begin{cases} x^2+y^2=4 \\ z=-1 \end{cases}$ and .

Answer: A circle of radius 2, lying in a plane one unit below the xy-plane.

12.2 Vectors in space

Addition



Note: $\vec{b}+(\vec{a}-\vec{b})=\vec{a}$

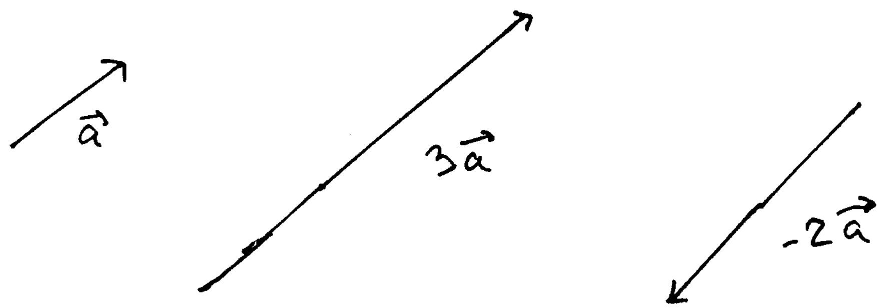
algebraically:

$\vec{a}=\langle 3,1,2\rangle \qquad \vec{b}=\langle 4,-3,5\rangle$

$\vec{a}+\vec{b}=\langle 7,-2,7\rangle$

$\vec{a}-\vec{b}=\langle -1,4,-3\rangle$

Scalar multiplication



$\vec{a} = \langle 3, 1, 2 \rangle$

$3\vec{a} = 3\langle 3, 1, 2 \rangle = \langle 9, 3, 6 \rangle$

$-2\vec{a} = -2\langle 3, 1, 2 \rangle = \langle -6, -2, -4 \rangle$

$\frac{1}{10}\vec{a} = \langle 0.3, 0.1, 0.2 \rangle$

Definition: $\vec{i} = \langle 1, 0, 0 \rangle$

$\vec{j} = \langle 0, 1, 0 \rangle$

$\vec{k} = \langle 0, 0, 1 \rangle$

ex: $3\vec{i} + \vec{j} + 2\vec{k} =$

$3\langle 1, 0, 0 \rangle + \langle 0, 1, 0 \rangle + 2\langle 0, 0, 1 \rangle$

$= \langle 3, 0, 0 \rangle + \langle 0, 1, 0 \rangle + \langle 0, 0, 2 \rangle$

$= \langle 3, 1, 2 \rangle = \vec{a}$

$= 2\vec{k} + 3\vec{i} + \vec{j}$

Defn: Length or magnitude of a vector:

If $\vec{a} = \langle a_1, a_2, a_3 \rangle$

the length of $\vec{a} = |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Remark In some textbooks the notation is $\|\vec{a}\|$.

ex: $\vec{a} = \langle 3, 1, 2 \rangle$

$|\vec{a}| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{9+1+4} = \sqrt{14}$

Properties of vectors: [See textbook]

12.3 Dot Product

Defn: If $\vec{a} = \langle a_1 \ a_2 \ a_3 \rangle$

and $\vec{b} = \langle b_1 \ b_2 \ b_3 \rangle$

then the dot product

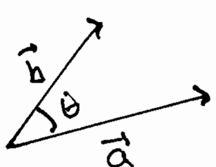
$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

ex: $\vec{a} = \langle 3, 1, 2 \rangle$

$\vec{b} = \langle 4, -3, 5 \rangle$

$\vec{a} \cdot \vec{b} = (3)(4) + (1)(-3) + (2)(5)$

$= 12 - 3 + 10 = 19$ A number NOT a vector!



Fact: $\boxed{\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta}$ where θ is the angle between $\vec{a}$ and $\vec{b}$.