Math 281

12.6 Quadric surfaces

Recall: A degree 2 polynomial equation in x and y

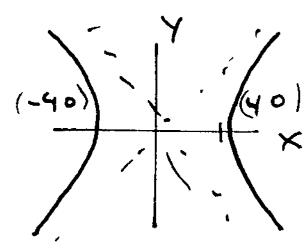
$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

will be some sort of conic section (or degenerate conic section): ellipse, parabola, or hyperbola.

ex: $x^2 - y^2 - 16 = 0$

$x^2 - y^2 = 16$

$\frac{x^2}{16} - \frac{y^2}{16} = 1$



Fact: A degree 2 polynomial in x, y and z will describe a quadric surface (or possibly a degenerate quadric surface). Up to rotation; translation there are six quadric surfaces:

1. Ellipsoids.
2. Hyperboloids of one sheet.
3. " " two sheets.
4. Cones.
5. Elliptic paraboloids.
6. Hyperbolic paraboloids.

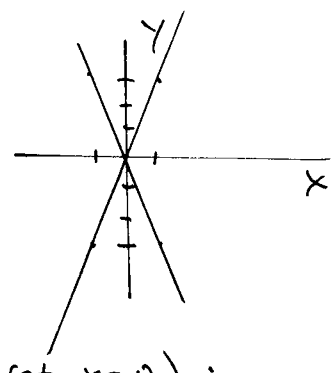
12.6 12) $9x^2 - y^2 + z^2 = 0$ can be written as

$9x^2 + z^2 = y^2$ Find the "trace" in the xy-plane by setting $z = 0$:

$9x^2 = y^2$

or $y = \pm\sqrt{9x^2}$

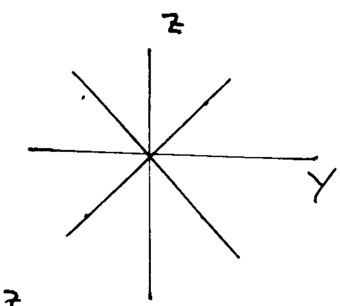
or $y = \pm 3x$



Find the trace in the yz-plane, (set $x = 0$):

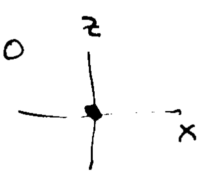
$z^2 = y^2$ or

$z = \pm y$



Trace in xz-plane? $y = 0$

$9x^2 + z^2 = 0$



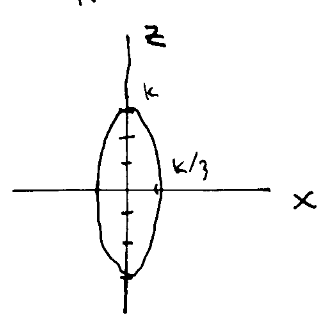
Trace in the plane $y = k$?

$9x^2 + z^2 = k^2$ or $\frac{9x^2}{k^2} + \frac{z^2}{k^2} = 1$

or $\frac{x^2}{(k/3)^2} + \frac{z^2}{k^2} = 1$

ellipse z-intercepts: $\pm k$

x-ints : $\pm \frac{k}{3}$



12.6 34) $4y^2 + z^2 - x - 16y - 4z + 20 = 0$

This seems to be an elliptic paraboloid like

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

$$4y^2 - 16y + z^2 - 4z = x - 20$$

$$4(y^2 - 4y + 4) + (z^2 - 4z + 4) = x - 20 + 16 + 4$$

$$4(y-2)^2 + (z-2)^2 = x$$

So, we could see (with more work) that this is an elliptic paraboloid with vertex at $(x, y, z) = (0, 2, 2)$ and open in the positive x-direction

Equivalent equation (divide by 4): $(y-2)^2 + \frac{(z-2)^2}{4} = \frac{x}{4}$

13.1 : Vector functions and space curves (all precalculus topics)

<u>main idea</u>: Vector-valued functions can always be interpreted as parametric equations, (and vice versa).

<u>Defn</u>: A vector function has the form

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$
$$= f(t)\,\hat{\imath} + g(t)\,\hat{\jmath} + h(t)\,\hat{k}$$

<u>ex</u>: $\vec{r}(t) = \langle 5\cos t, 3\sin t \rangle \qquad 0 \le t \le \pi$

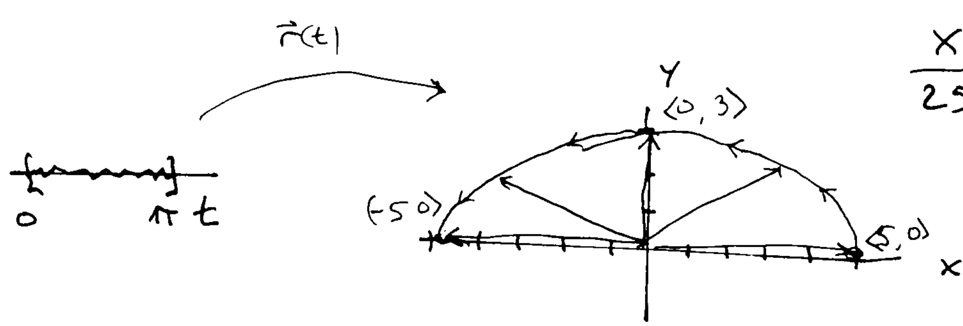
t	$\vec{r}(t)$
0	$\langle 5, 0 \rangle$
$\pi/4$	$\langle 5/\sqrt{2}, 3/\sqrt{2} \rangle$
$\pi/2$	$\langle 0, 3 \rangle$
$3\pi/2$	$\langle -5/\sqrt{2}, 3/\sqrt{2} \rangle$
π	$\langle -5, 0 \rangle$

<u>Remark</u>: This can be thought of as a set of parametric equations.

$$\begin{cases} x = 5\cos t \\ y = 3\sin t \end{cases}$$

Eliminate the parameter: $\frac{x}{5} = \cos t \Rightarrow \frac{x^2}{25} = \cos^2 t$

$\frac{y}{3} = \sin t \qquad \frac{y^2}{9} = \sin^2 t$



$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

ex: Identify and sketch the curve whose equation is

$$\vec{r}(t) = \langle 3\cos t,\ 3\sin t,\ t \rangle$$

t	$\vec{r}(t)$
0	$\langle 3, 0, 0 \rangle$
$\pi/2$	$\langle 0, 3, \pi/2 \rangle$
π	$\langle -3, 0, \pi \rangle$
$3\pi/2$	$\langle 0, -3, 3\pi/2 \rangle$
2π	$\langle 3, 0, 2\pi \rangle$

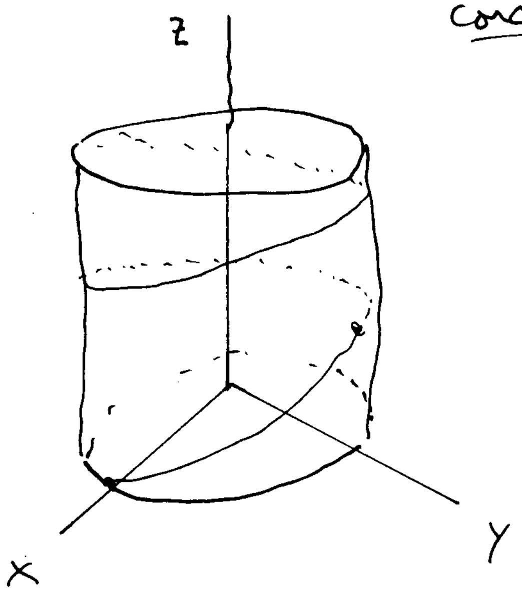
Equivalent parametric eqns.

$$\begin{cases} x = 3\cos t \\ y = 3\sin t \\ z = t \end{cases}$$

So $\quad x^2 + y^2 = 9\cos^2 t + 9\sin^2 t$

or $\quad x^2 + y^2 = 9(\cos^2 t + \sin^2 t)$

or $\quad x^2 + y^2 = 9$



<u>conclude</u>: whatever curve we're tracing, it lies on a cylinder of radius 3, and axis = z-axis.

<u>Equivalently</u>: the projection of the curve onto the xy-plane is a circle of radius 3.

<u>Another way</u> to eliminate the parameter:

$$\begin{cases} x = 3\cos t \\ z = t \end{cases} \Rightarrow x = 3\cos z$$

$\uparrow$
a generalized cylinder