

Comparing Different Numeration Systems

Unique feature in the Hindu-Arabic system

If you pay attention to all numeration systems we have studied, you will notice that only the Hindu-Arabic system has different symbols for each number less than the base. In other words, there are different symbols for one, two, three, ... up to nine because the base is ten.

In the Egyptian system, the base is also ten, but they did not have a unique symbol for two. They used the symbol Staff (standing for one), twice. To represent three, they would use the same symbol three times and so on.

In the Babylonian system, the base is sixty, but there were only two symbols, ▼ and ◀ (even though later on they added a place holder †). To represent thirty-two, which is less than the base, they had to repeat each symbol several times namely, ◀◀▼▼.

The Roman and Myan systems are even more inferior and more difficult to use.

One disadvantage in using several symbols for a number less than the base is that the number of symbols in the expression is not proportional to the value it represents. In other words, more symbols may not represent a larger number. For example

	<u>one hundred</u>	<u>seven</u>
Egyptian numerals	⓪	
Babylonian	▼◀◀◀	▼▼▼▼▼
Hindu-Arabic	100	7

It is clear that in the Hindu-Arabic system, the more symbols (including the place holder) used in a numeral expression, the larger the number it represents. This unique feature will make comparison of numbers more visual – the bigger the expression, the larger the number.

Another disadvantage is best illustrated in the Babylonian system. Even though it also use place value, the place value of a symbols in a particularly fixed position of an expression is not always the same. For instance, the place value of the second symbol can sometimes be sixty and sometimes be one,

▼◀▼	◀▼▼
the middle symbol	the middle symbol
has place value sixty	has place value one

(whereas in the Hindu-Arabic system, the place value of the second symbol from the right is always ten.)

The consequences of this drawback are

1. the value of an expression may be misinterpreted, for example ▼▼ can be interpreted as two and can also be interpreted as sixty + two. The existence of place holder does not help as there is no empty places in between. The Babylonian seemed to notice this and they tried to distinguish these two by putting a larger gap between the two triangles if they have different place values, such as ▼ ▼ means sixty-one and ▼ ▼ means two. However this is not very satisfactory as different people have different standard for gaps.

- there will not be any easy written algorithms for addition and multiplication etc. because they cannot line up the symbols column by column such that in each column all symbols have the same place value, for example

$$\begin{array}{r}
 \triangleleft \triangleleft \triangleleft \\
 + \quad \triangleleft \triangleleft \\
 \hline
 \end{array}$$

we cannot add the symbols column by column as we used to do in the Hindu-Arabic system.

Advantage of using place value

The most important reason for using place value is that a finite set of symbols can represent an infinite set of numbers, all it needs is a longer string of symbols for a larger number. In other words, there is no need to introduce a new symbol when a larger number is encountered.

Note: the use of place value does not necessary make the numeric expression more compact, as can be seen in the Babylonian system. The only way to make an expression compact is to use a unique symbol for each number less than the base.

Disadvantage of using place value

The Hindu-Arabic system is compact and efficient but difficult to learn. It may not be obvious to a person who has learned it and has been using it for many years, however for children or someone who come across with it the first time may find the following points difficult to master.

- The order of the symbols appear in a numeric expression is very important, 583 is different from 835. To find out which one is larger also require more knowledge of the place value than just recognizing the symbols.
- It requires memorization and mental activity to interpret the value represented by a numeral, for example, to find the value represented by 201478 one has to remember that the place values from right to left are one, ten, hundred, thousand and so on, and then match the symbols with their corresponding place values to get the answer of two hundred and one thousand four hundred and seventy eight.

The most common mistake for beginners is to simply add the values of the symbols together such as misinterpreting 111 as three. One possible explanation for this is that children learn counting first and in counting objects, they group like objects together. As they see three identical symbols in 111, their natural response would be the value three.